Math Education in the US: Still Crazy After All These Years

A presentation by Barry Garelick;
ResearchED; Oxford, UK
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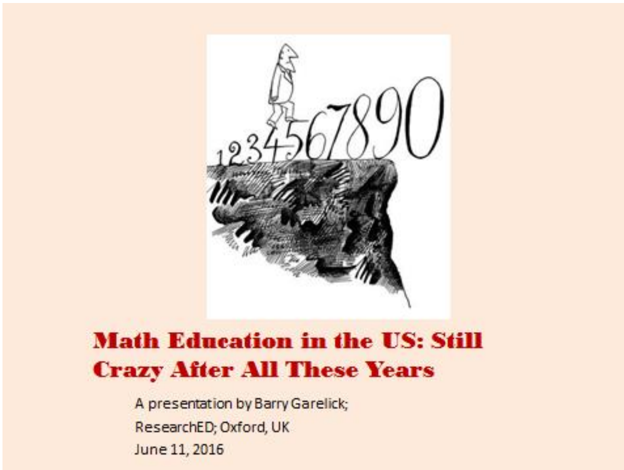
Barry Garelick:

- Currently on a second career as a math teacher, having recently retired from the federal government.
- Majored in math at University of Michigan
- Fully certified to teach math in California.
- For the last 4 years has been teaching math
- Researched math education during a 6 month assignment to a US Senator in Washington DC
- Over the past 10 years, has written several books, and articles about math education focusing on the evidence behind effective math instruction, which have appeared in the Atlantic, Education Next, and AMS Notices.

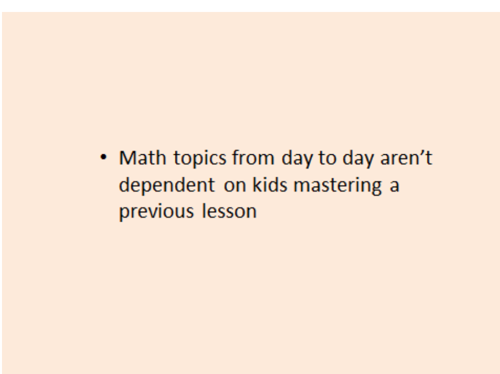


The title of my talk is the title of my latest book –Math Education in the US: Still Crazy After All These Years. I was reticent to do this, not only because of shameless promotion of my book, but for what appears to be a US-centric look at math education. The arguments between traditionalists and progressives in the UK on how best to teach math, parallel those in the US (and Canada), however, so I thought how we're dealing with it might be of interest.

A brief note on some terminology: when I refer to math reformers, it means people who embrace a progressivist ideology. I use a combined form, like progressivists/reformers.

Before I get into it, here's how I got into the royal mess known as math education. I first became aware that something was off with math education when my wife and I met with our daughter's second grade teacher, concerned that our daughter wasn't learning her addition and subtraction facts.

We were told not to worry because kids eventually "get it" and then given this good news:



I was willing to suspend my disbelief, being new to all this and thinking that perhaps there were better ways of doing things than when I was in school.

But over the next few years I was disabused of that idea as I learned about the reform/progressivist approach to math: in which students discover and teachers facilitate; in which students must work cooperatively in groups; where students must provide explanations for problems so easy they defy explanation, where consistently getting right answers to varied problems does not signify "understanding". In short, it is an educational orientation that I and others like me 1) did not believe in and 2) found ourselves immersed in.

In light of all this, I decided to teach math when I retired. I attended night classes at an education school and began writing articles about the state of math education in the US.

My articles, while supported by many parents who shared similar concerns as mine for their childrens' educations, have also been met with opposition from teachers who have been steeped in the group-think of progressivist-dominated education schools.

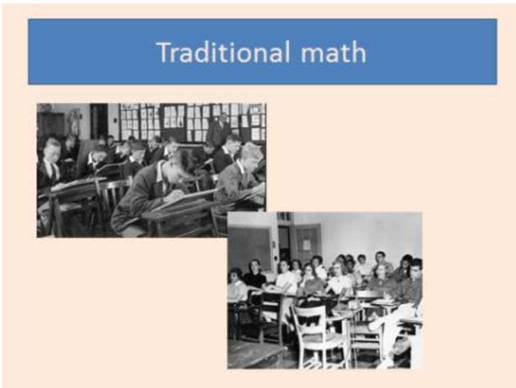
And with the adoption of a set of standards by 42 states called Common Core the ideological battle in the US has become even more pronounced. But I don't want to get too far ahead of myself. So -- spoiler alert—this headline, while made-up, tells a disturbingly accurate story.

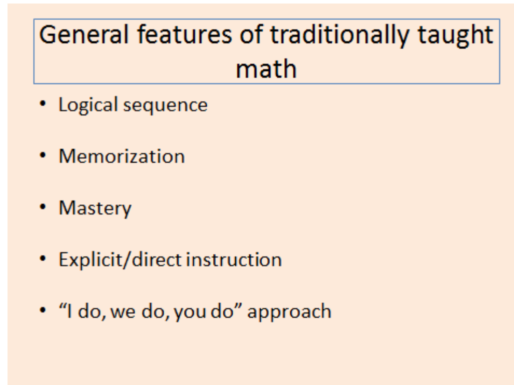
My talk today is the backstory to this blunt statement, and is an overview of the state of math education in the US.

Types of Math Education

- Traditional or conventionally taught math
- Reform/Progressivist math
- Common Core
- Guardedly optimistic statements

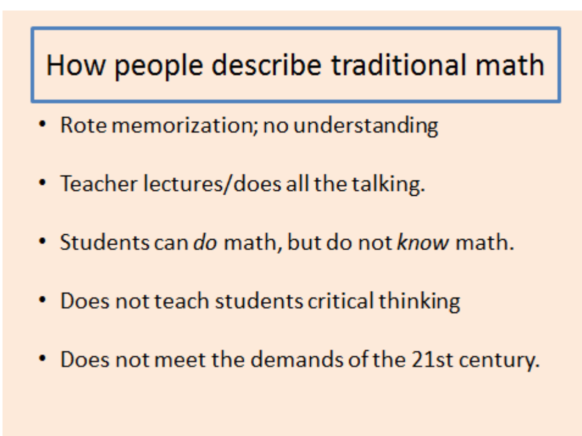
Let’s look first at what is known as “traditionally taught math”.





Traditionally taught math in general means covering topics in a logical sequence, requiring the memorization of key facts and mastery of basic procedures which are then built upon over time. The teaching method and classroom format presents information using explicit and direct instruction in a “whole class” manner. Usually taking the form of “I” demonstrate the technique, “we” now try it together, and now “you” do these problems.” I say “general features” because there are variations, with some discovery learning inherent in it, Socratic type discussions, group work, and activities that build engagement.

But traditional math is frequently mischaracterized as teaching that is inherently poor, stilted, dull, and artificial:



Anyone challenging these mischaracterizations by saying that they seemed to do all right with it, is told “You’re the exception”

This mischaracterization and finger-pointing denigration of past programs has been a strategy for a long time and could probably be tracked all the way back to Euclid. Not having that kind of time, I didn't go back that far; I stopped at 1952, at the intro to a 4th grade arithmetic textbook.

Old vs New

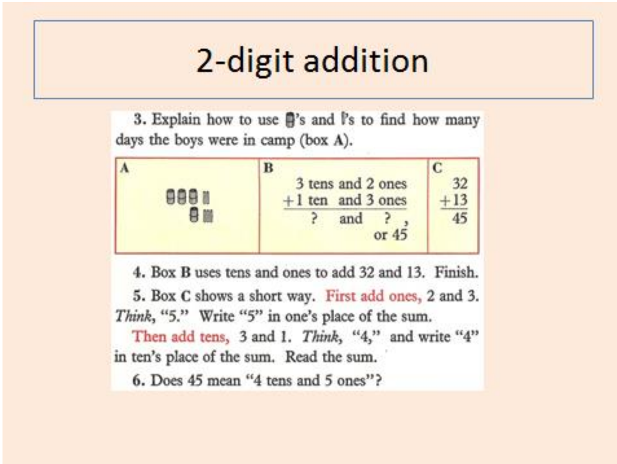
Area of Comparison	The Older Program	The Newer Program
Arithmetic content	Taught as facts, skills and habits of procedure	Facts and skills developed after understanding.
Emphasis in classroom teaching	Oral and written drill	Drill for computational competence follows understanding.
Basis for evaluating pupil progress	Rate and accuracy in computation and problem solving.	Flexibility of thought processes.

Source: Growth in Arithmetic; Grade 6 (Teacher's Edition); Clark, Junge and Moser (1952)

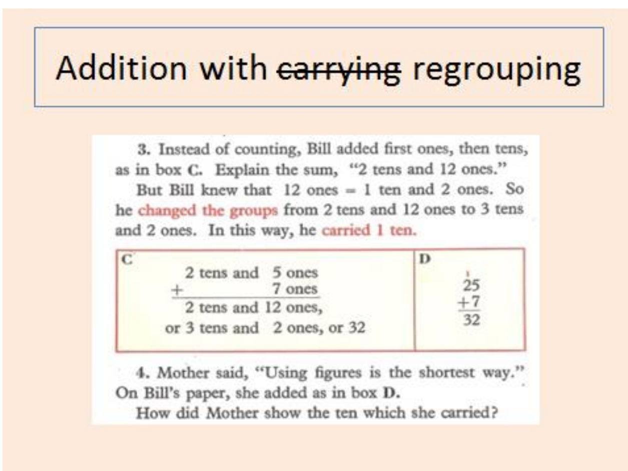
The authors of the 1952 textbook disparage previous approaches as teaching facts, skills and procedures with no understanding, and mechanized drills. This chart could just as well be from one of today's textbooks. Notice the "Facts and skills developed after understanding", something we'll come back to later.

The people who wrote the math books from the 1930's through the 1960's were the reformers of their day, and they argued for the same things as today's reformers. But their approach was different—and some (including me) have argued that it was effective.

One advantage of being as old as I am is that I can remember how math was taught. My era was 50's and 60's The elementary arithmetic books I had were written by some of these reformers—notably William A. Brownell who is thought highly of even by today's reformers—including the education critic Alfie Kohn. Let's take a look:

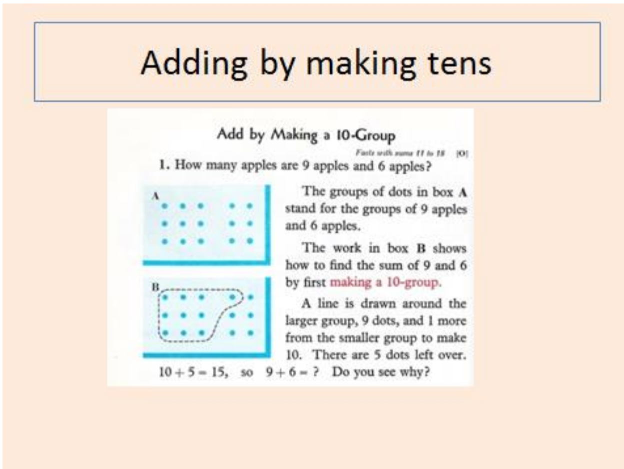


Two-digit and addition and subtraction was explained in terms of pictures, words and finally, algorithm.

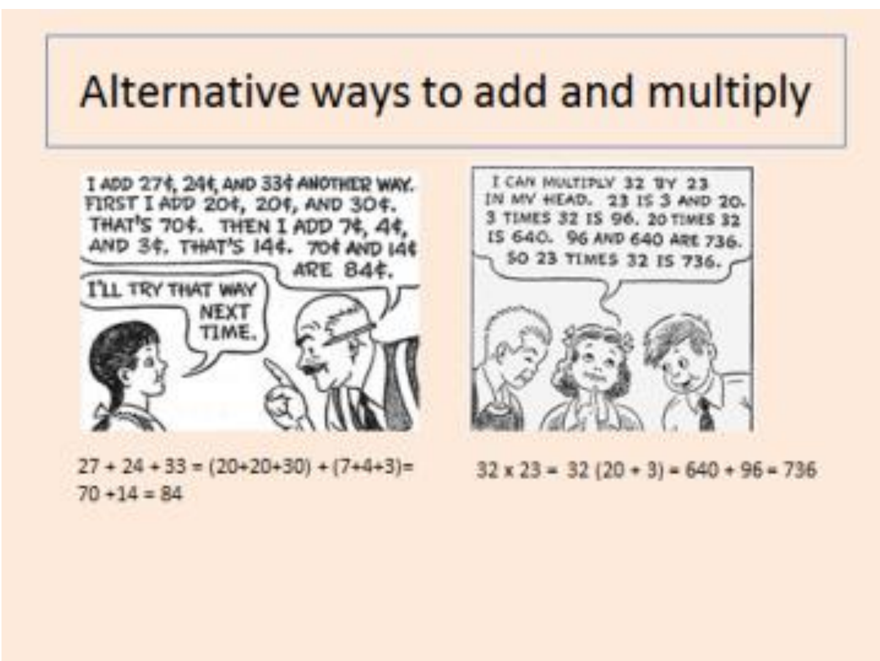


Learning how to "carry" (now called "regrouping") came shortly after. And even if you didn't fully understand why the standard algorithm worked for problems where you didn't need regrouping, when it came to problems where you needed it, it became very obvious. Here is a clear explanation of conceptual underpinning as well as the procedure. In fact, it would be pretty hard to teach this *without* explaining what place value is since at every stage you have to reference it.

Similarly, multiplication and division were also explained and not, as frequently mischaracterized, taught as times tables to be memorized with no connection or understanding of what multiplication or division meant.



Mental math techniques were also taught. One technique is known as the "making tens" method of addition. Here it shows how 9 + 6 can be represented by taking a 1 away from the pile of 6 dots to add to the 9, to make 10. Then adding the (6-1) or 5, to get 15.



There were also strategies for two digit addition and multiplication—these came from a 5th grade textbook—not mine; this one was from 1948. The addition relies on place value, and the multiplication on decomposing tens and the distributive property. Mental math, therefore, is not something that is new, but has been around for quite a while.

There are many more examples including explanations of fraction operations and why/how they work. It wasn't rote memorization and topics were definitely connected—and built upon. Could it have been better? Of course. There could have been more challenging and interesting problems for one thing. But yesterday's students when entering algebra had a far superior grasp of the basics than many of today's students, and were well prepared for the next step.

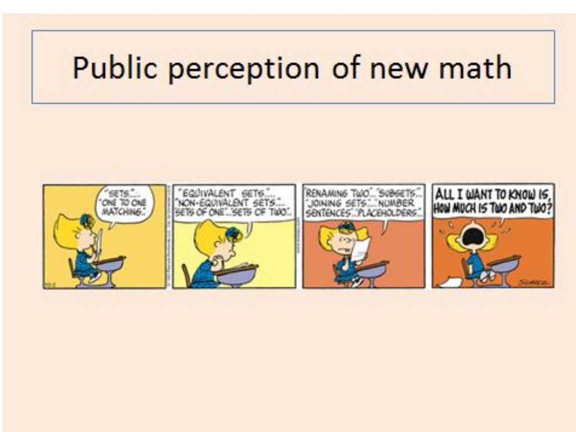
Now there are two things I must mention very briefly before I turn to reform/progressivist math:

First, on October 4, 1957, the Soviet Union launched Sputnik. This event caused overall panic and the US felt we needed to boost science and mathematics education or we would fall further behind the Soviet Union.

The result was the typical time-honored solution that governments take when faced with a problem; they threw money at it and a program known as the "new math" was born.

The curriculum, designed primarily by mathematicians, was fairly effective for high school math topics, but K-6 math used a set-theoretical approach that was too formal for many students—and teachers.

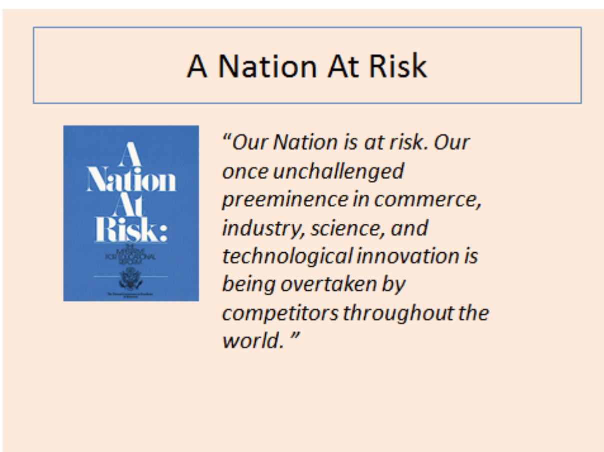
A Peanuts cartoon from the mid-60's shows how it was viewed by the public.



The new math also incorporated aspects of "discovery" and was often billed as "A whole new way to teach math" and in some ways set the stage for that next phase of reform math which emerged in the 90's.

"A whole new way to teach math."

New Math was deemed a failure in the early 70's and math returned to a "back to basics" approach. Some say a bit too much, but students did improve in computational math.

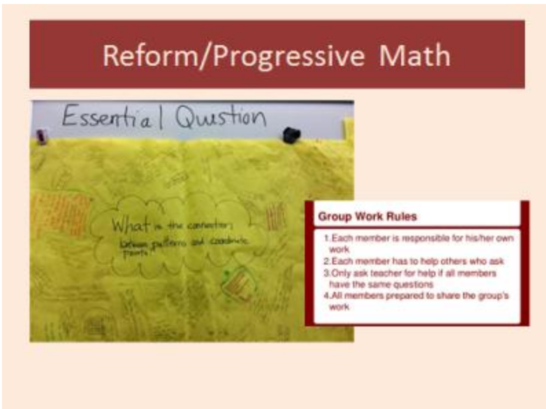


In 1983 another key event took place. What with the US lagging economically behind other countries, and the cold war still going on, panic set in again. The National Commission on Excellence in Education published *A Nation at Risk*, warning that it we didn't get our act together educationally we would fall behind.

This time, an organization called National Council of Teachers of Mathematics or NCTM responded to the crisis and in 1989 published a set of math standards, (revised in 2000) to serve as guidance to our education system. National Council of Teachers of Mathematics was founded in 1920 and is one of the largest private organizations concerned with mathematics education. *The*

Curriculum and Evaluation Standards for School Mathematics, purported to put the country back on the math track.

With that, the next era in math education was ushered in.



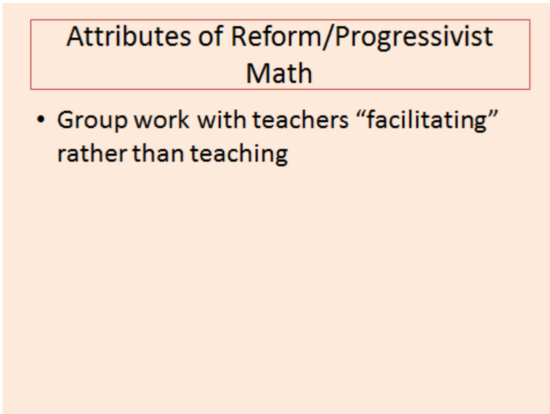
Following the time-honored tradition of disparaging everything that came before, NCTM promoted its progressivist/reform standards as a "whole new way of teaching math".

Key provisions of NCTM's standards

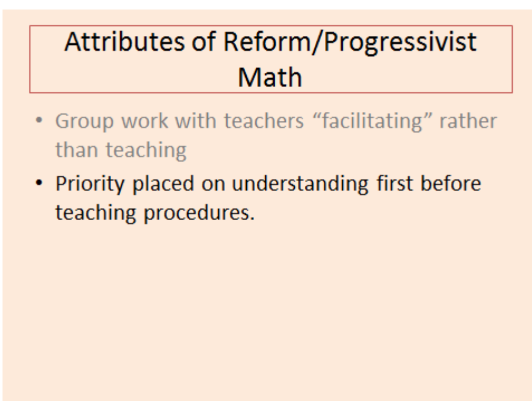
- Emphasized the use of calculators in all grades.
- Decreased attention to:
  - "complex paper-and-pencil computations,"
  - "long division,"
  - "paper-and-pencil fraction computation,"
  - "rote memorization of rules,"
  - "direct instruction/whole class teaching"

NCTM's standards specified content that was to be taught, but the areas in NCTM's standards that caught the attention of reformers/progressivists are shown here. The advent of the calculator made doing many so-called paper and pencil operations seem unnecessary—and supposedly freeing up time developing "understanding".

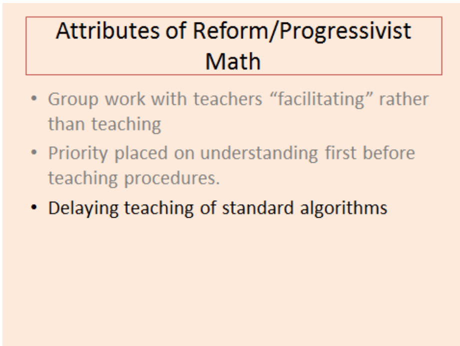
These standards were not any kind of state requirement, but some states modeled their standards after NCTM's, and education schools promoted the ideas in their classes. While traditional math teaching hasn't disappeared, progressivist math teaching practices over the past 25+ years have been growing slowly and steadily. We have been seeing:



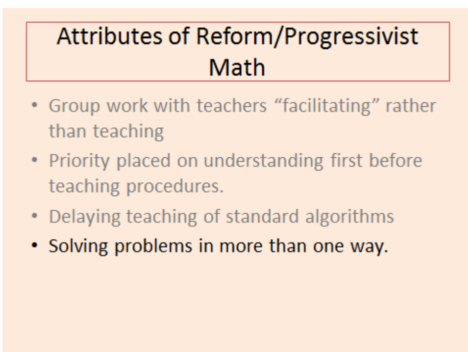
More group work in which students supposedly teach each other, with the teacher "facilitating" as a guide on the side, as students engage in discovery learning. The "I do, we do, you do" technique is considered "rote memorization".



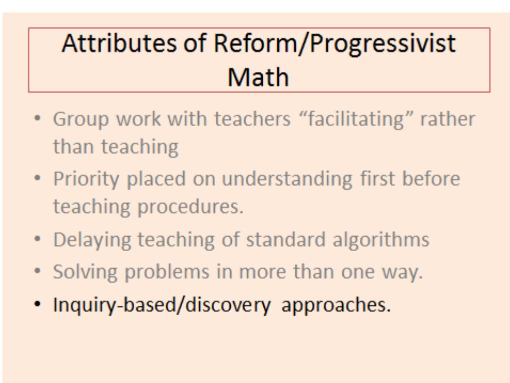
Understanding and procedure work in tandem—you need both. Understanding sometimes comes first, sometimes later. Traditional math instruction devotes instructional time to conceptual understanding and procedures, but conceptual understanding now was given priority under an "understanding first, procedure later" approach.



The teaching of standard algorithms was delayed. Students were required to do alternative approaches first. Teaching the standard algorithm first was viewed as eclipsing understanding of these procedures. Side dishes now became the main course and students grew confused—sometimes profoundly so.

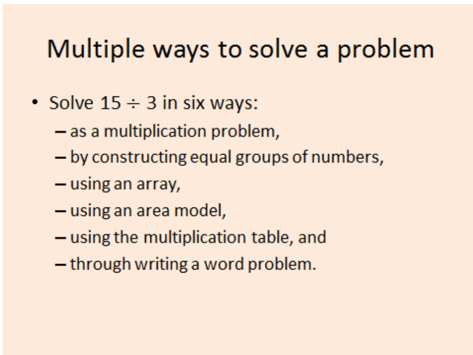


Problems are to be solved in more than one way, in the belief that doing so imparts and provides evidence of "understanding".

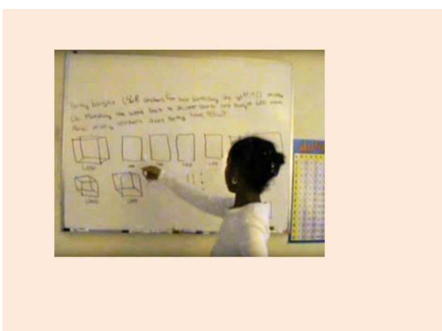


Inquiry-based and discovery approaches thought to result in greater understanding.

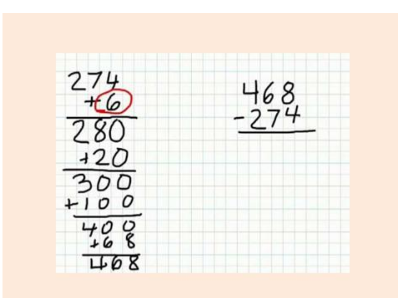
As a result:



You have students being required to solve simple problems in multiple ways supposedly to enhance discovery and impart understanding.



You have students drawing pictures for much longer than necessary, serving as both a means to simultaneously understand and explain an otherwise simple procedure.



The "counting up" method for subtraction. (Same process used for making change in the days when cash registers didn't have calculators built in, and cashiers had to "count up" to make change.

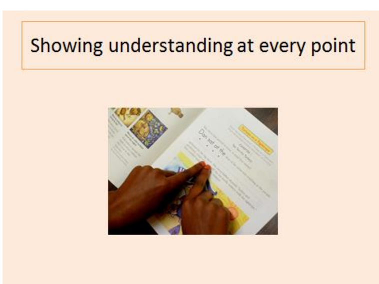
Adding tens first method

- 19 + 25= ?
  - ✓10 + 20 = 30
  - ✓9 + 5 = 14
  - ✓30 + 14 = 34

The tens and ones added separately method which was shown earlier; tens and ones are added separately and the totals added together.

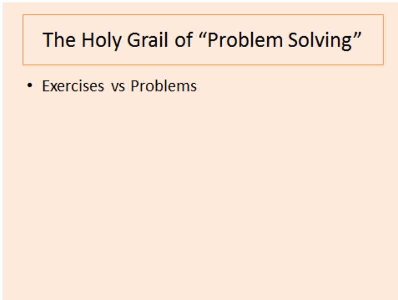
Many of these are the same methods for addition and subtraction that are taught in traditional math. But *AFTER* students achieve mastery with the standard algorithms—and not for weeks on end. It is clear what the main dish is, and what the side dishes are. Alternative procedures learned *after* mastery of the standard algorithm actually helps to explain how the standard algorithm works.

But progressivists want to teach for "understanding". They do so without defining what understanding is, but believe that the alternative approaches provide the conceptual underpinning. But drawing pictures and providing awkward explanations is not a sign of, nor path to understanding. It is an insistence on students articulating "meaning" out loud at every stage.

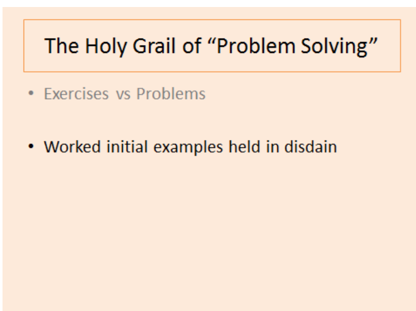


It is the arithmetic equivalent of forcing a reader to keep his finger on the page sounding out every word, with no progression of reading skill. It amounts to little more than a "rote understanding" for procedures students probably can't perform for problems they cannot solve.

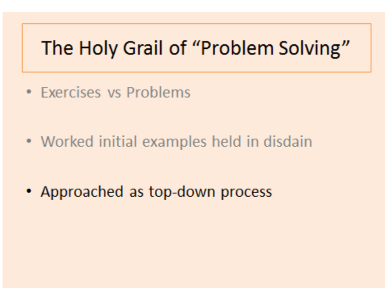
Which brings us to the holy grail of reform/progressivist math: Problem Solving.



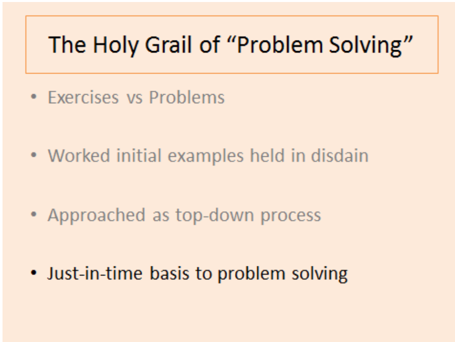
The term "problem solving" has acquired its own meaning and usually entails discovery learning. Routine problems are viewed as "mere exercises", which involve applying algorithms, memorizing a solution method, or mnemonic device. The goal is for students to solve problems they've never seen before.



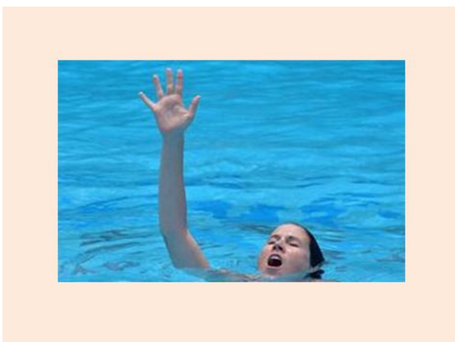
Bottom-up approaches, like worked examples, are held in disdain. Progressives/reformers strive for "authentic" problems that are "application-based" and engaging. The reformers who are adept at problem solving seem to have forgotten that the process of working standard problems is what got them to where they are.



Problem solving then is often approached as a top-down process where students are given problems in entirely new contexts with little or no prior knowledge—math problems that require outside-of-the-box insight and/or inspiration are thought to impart "understanding" and produce a problem solving schema independent of the mathematics and techniques that would allow students to solve the problems in the first place.



In the top-down approach, problems may be open-ended or they may require specific mathematical skills and procedures, many of which students have yet to learn. Rather than mastering these things beforehand, they struggle to learn what is needed on a just-in-time basis—sometimes referred to as "problem based learning".



This is like teaching someone to swim by throwing them in the deep end of a pool and telling them to swim to the other side. The teacher shouts from the side of the pool: "Now would be a great time to learn the breast stroke" and shouts the instruction to the struggling student. While an expert swimmer may struggle to perfect a swim stroke, a novice struggles to keep from drowning—a struggle that doesn't teach them how to swim.

Which brings us to the Common Core Standards --grade-by-grade guidelines written for both math and English language arts. Written by Council of Chief State School Officers and NGA. They have been adopted in 42 states and DC, and replaced the previous state standards in math and ELA—with the objective that the entire country teach to the same goals. They were published in 2010 with extremely little public input and amidst much controversy.

I would be remiss if I didn't mention that there are good things in these standards—requirements for fluency and mastery of standard algorithms. Also some curricula/textbooks interpret CC in sensible effective ways—Singapore Math series being one of them. There are also teachers who teach in sensible ways—if they are allowed to, that is.

Having said that; from what I see happening Common Core is the gasoline on the ideological fire of the reformers/progressivists which has been burning slowly and steadily for the past 25+ years in the US. Maybe I'm being over-dramatic here. After all, they call for demonstrated fluency in the math facts, AND, learning the standard algorithms.

"These Standards do not dictate curriculum or teaching methods."

And there is no mandate anywhere dictating that teachers must use discovery, or student-centered approaches. It even says so!

The CC math standards are promoted as 'pedagogically neutral', as 'guidelines, not a curriculum' and 'Teachers can use whatever tools they want to help students meet the standards'. Why is it, then, that many Common Core inspired assignments shown on television or the internet, bear the reform/progressivist imprints: student-centered and discovery-driven assignments; group-based and real-life-relevant; touted as fostering 'critical-thinking'?

Getting the reform/progressivist message across

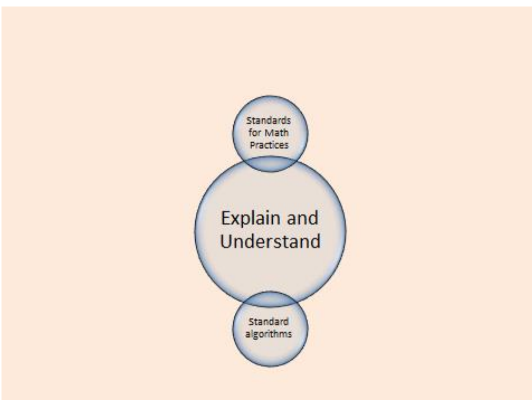
One clue comes from the language embedded in the standards: subtle "nudge-nudge wink-wink know what I mean" references that are picked up by reformers/progressivists.

Explain and Understand

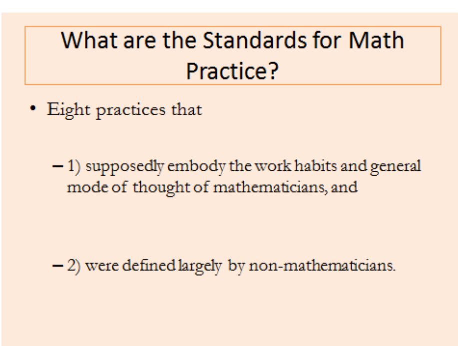
- "...*explain* the reasoning used." (2nd year/1st grade)
- "*Explain* why addition and subtraction strategies work, using place value and the properties of operations. (3rd year/2nd grade)
- "*Understand* the relationship between numbers and quantities... (K)
- "*Understand* a fraction as a number on the number line..." (4th year/3rd grade)

The words "explain" and "understand" are the prime examples of this, which are embedded in many of the standards. These are key signals to the reformers/progressivists. And given the progressive/reform background of some of the people on the math stds team, I do not believe this is entirely coincidental.

The three lead writers were Phil Daro, Bill McCallum and Jason Zimba. Phil has a degree in English literature, and I'm told has a minor in math, and taught high school algebra, briefly. He has been active for years in promoting reform math. Bill McCallum, a math professor who teaches at University of Arizona has been sympathetic with the reform approach. Jason Zimba who has a doctorate in Physics and taught at Bennington College is less sympathetic to reform math ideas than the other two; but public statements he has made indicate he is not averse to such ideas. The rest of the writing team as well as the team that reviewed and commented on the standards, were largely reform oriented. Jim Milgram, a retired math professor from Stanford, who was on the review committee refused to sign off on the final package.

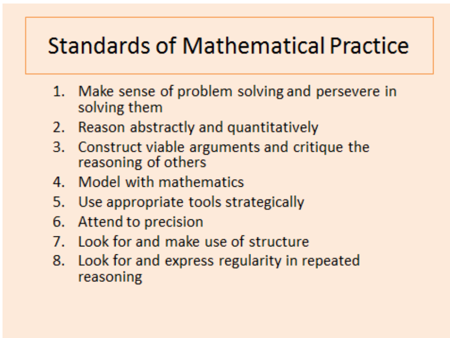


The wink winks of "understand" and "explain" feed into 2 key components of Common Core: 1) A set of 8 standards called "Standards for Mathematical Practice" or SMP's and 2) the standard algorithms. I'll talk about the SMP's first.

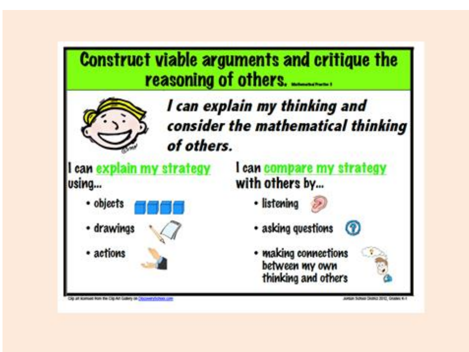


The SMPs are eight practices that 1) supposedly embody the work habits and general mode of thought of mathematicians, 2) were defined largely by non-mathematicians. They also were based on what were called "process standards" in NCTM's standards and were one of the main mechanisms for putting into action reform math practices. So already there's a built in nudge nudge/wink wink.

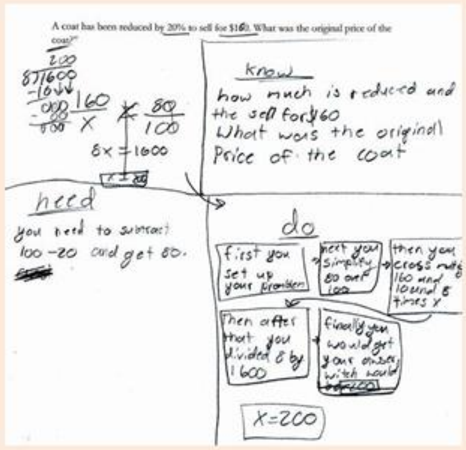
And here they are:



Taken at face value, they are seemingly benign. For example, there's nothing wrong with the first point: "Make sense of problem solving and persevering in solving them." Who wouldn't want this? But the words "problem solving" are a signal for convoluted and tedious "real world" problems and "just in time" learning. The SMPs are thought to teach "habits of mind" often outside of the context of the math courses from which they would arise naturally.



They don't have to be interpreted this way. But the nudges and winks have reformers/progressivists stating that CC in general and the SMP's in particular require inquiry activities and collaborative group work, students solving problems in more than one way and most importantly for "explaining their answers" rather than simply showing their work.



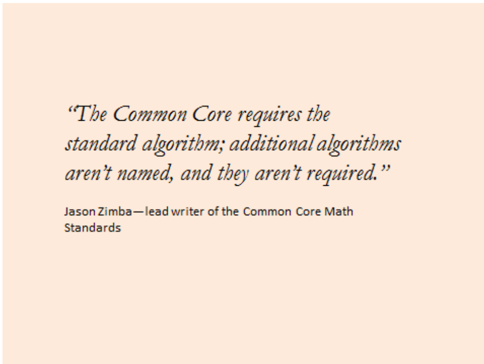
For this last, the assumption is if students cannot explain their reasoning, in writing or otherwise, students lack understanding. But since students in lower and middle grades are novices and lack skill in articulating ideas, such explanations often have little mathematical value. It often amounts to students engaging in the exercise of guessing (or learning) what the teacher wants to hear. More "rote understanding" via "rote explaining"

Which brings me to how CC handles the standard algorithms. The reformers/progressivists believe that teaching standard algorithms first eclipses students' understanding of why the procedure works.

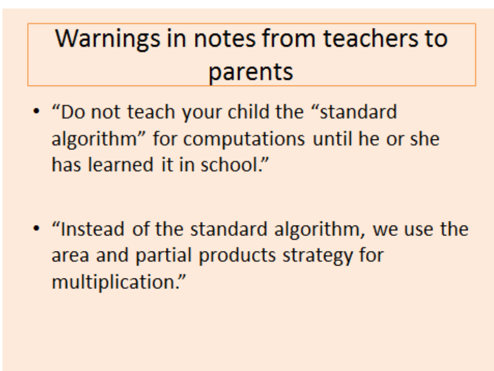
Standard algorithms—after alternatives

- Grade 4 (Yr 5) : "Fluently add and subtract multi-digit whole numbers using the standard algorithm."
- Grade 1 (Yr 2) grade: "Add within **100…using concrete models or drawings and strategies based on place value"**
- Grade 2/3 (Yr 3/4): "Fluently add and subtract within 100/1000 **using strategies based on place value"**

In the standards, the first mention of a std algorithm is in the 4th grade (yr5)—for multidigit addition and subtraction. (A delay also happens with multiplication (delayed to 5th/yr 6) and long division (delayed until 6th/Yr 7). Prior to that, the standards refer to drawings/strategies based on place value.

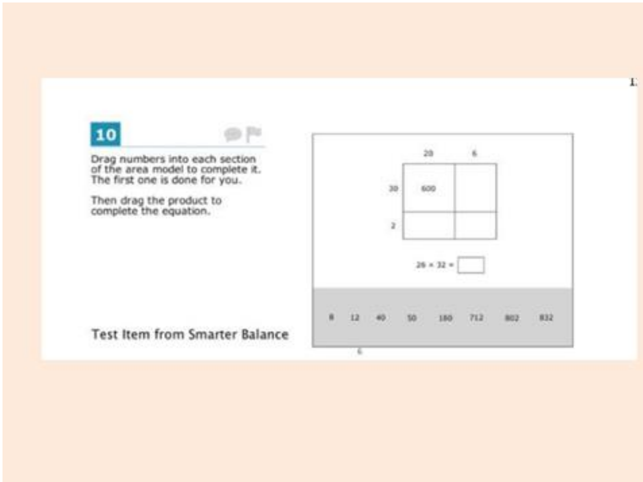


But, according to two of the lead writers of the standards, Jason Zimba and Bill McCallum, the standard algorithms can be taught earlier than the year in which they appear. Zimba in fact says this in writing, and recommends teaching it in 1st grade/Yr 2. But the built-in delays are a very big nudge-wink to reform/progressivist practices, which may be why despite Zimba's statement, the std algorithms are delayed, in the belief students are better able to conceptualize the "why" instead of the "how" in performing operations and explain what it is they're doing.



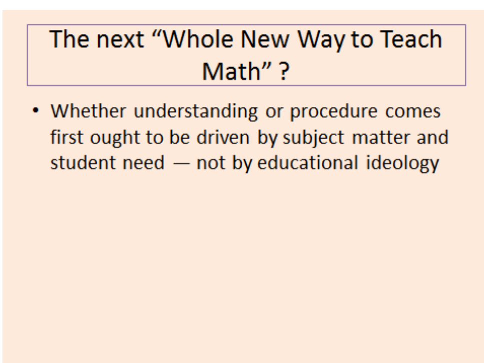
Word is apparently not getting out. Teachers have been sending notes to parents telling them *not* to teach them the standard algorithms at home.

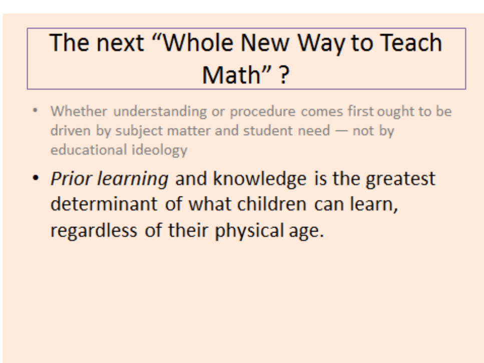
But teachers who may use (or who are allowed to use) the more traditional techniques will still have to deal with the elephant in the room: the standardized tests given at the end of the year. The Common Core-aligned tests given in the US require some facility with alternative strategies and some written explanations.



As shown here on a sample test question, students must know how to multiply two digit numbers via the area model.

I don’t wish to close on such a sour note, so as promised, here are some guardedly optimistic closing thoughts. The Common Core standards may be a blessing in disguise. The reform methods and philosophy are getting more notice – and protest-- than ever before. With the increased attention and pressure perhaps the progressivist philosophy will lose its grip and math education will be approached more effectively. Here is what I and many others would like to see become part of the next version of “A whole new way to teach math” and certainly not limited to the US:





The next "Whole New Way to Teach Math" ?

- Whether understanding or procedure comes first ought to be driven by subject matter and student need — not by educational ideology
- *Prior learning* and knowledge is the greatest determinant of what children can learn, regardless of their physical age.
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- Mistakes should not be clung to just because of the time spent making them.

QUESTIONS

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